

# A Dual Characterization of Pareto Optimality

Aldo Montesano

**Abstract.** According to Pareto's definition an allocation is Pareto optimal if there is no other feasible allocation that is at least as good for all individuals and better for some individuals. This definition of Pareto optimality is currently adopted in economics as a condition of efficiency, *i.e.* no waste of resources. Effectively, Pareto derived the analytical condition for optimality with a reasoning linked to saving resources. The correct form of this reasoning was introduced by Allais with the notion of maximal distributable surplus: an allocation is efficient if the utilities obtained by the individuals with this allocation cannot be reached with a feasible allocation that consumes less resources. Consequently, the relation between maximization of utilities and minimization of resources has to be taken under consideration. This paper introduces and examines the duality between these two problems. Under a weak assumption of monotonicity (between the available amount of resources and the feasible maximal utilities), equivalence is established between 'Pareto maximal allocations' (*i.e.* the allocations which generate a utility vector that is not dominated by the utility vector generated by any other feasible allocation) and 'Pareto efficient allocations' (*i.e.* the allocations which generate a utility vector that cannot be generated by any other feasible allocation which requires a smaller resource vector than the resource vector required by the allocation under consideration). In particular, the duality between utilities and resources allows the social loss caused by inefficiency to be measured in terms of a quantity of resources. Moreover, the introduction of the 'social welfare function' and its dual concept, 'the social requirement function', allows the extension of the Gossen-Menger loss principle to the whole economy. Finally, the implications of duality between utilities and resources are also considered in regard to the notion of Potential-Pareto-Dominance, *i.e.* to the comparison of different situations with the same consumers.